



第8讲：三角函数公式化简 + 题型拓展 + 技巧衍生 (1)

题型一：定义+同角关系

1. (2023·乙卷) 若 $\theta \in (0, \frac{\pi}{2})$, $\tan \theta = \frac{1}{3}$, 则 $\sin \theta - \cos \theta =$ _____.

2. (2025·松江三模) 设定义在区间 $(0, \frac{\pi}{2})$ 上的函数 $y = 6\cos x$ 的图像与 $y = 5\tan x$ 的图像交于点 P , 过点 P 作 x 轴的垂线, 垂足为 P_1 , 直线 PP_1 与函数 $y = \sin x$ 的图像交于点 P_2 , 则线段 P_1P_2 的长为 _____.

3. (2024·甲卷) 已知 $\frac{\cos \alpha}{\cos \alpha - \sin \alpha} = \sqrt{3}$, 则 $\tan(\alpha + \frac{\pi}{4}) =$ ()

A. $2\sqrt{3} + 1$

B. $2\sqrt{3} - 1$

C. $\frac{\sqrt{3}}{2}$

D. $1 - \sqrt{3}$

4. (2025·河南模拟) 设 $\tan \theta = \frac{1}{2}$, 则 $\frac{\tan \theta + \sin \theta \cos \theta}{\cos 2\theta} =$ ()

A. $\frac{1}{2}$

B. 1

C. $\frac{3}{2}$

D. 2





5. 已知 $\theta \in (0, \frac{\pi}{4})$, 则 $\frac{\sin^4\theta + \cos^4\theta}{\sin^3\theta\cos\theta - \sin\theta\cos^3\theta}$ 的最大值为 ()

- A. 4 B. 1 C. $-\sqrt{3}$ D. $-2\sqrt{2}$

6. (2025·湖南三模) 已知 $\sin\alpha + 2\cos\alpha = -1$ ($\cos\alpha \neq 0$), 则 $\tan 2\alpha =$ ()

- A. $-\frac{24}{7}$ B. $-\frac{17}{31}$ C. $\frac{17}{31}$ D. $\frac{24}{7}$

7. (2025·安徽模拟) 已知 $a = \frac{5}{4}, b = \sin\frac{5}{4}, c = \tan\frac{5}{4}$, 则 ()

- A. $c < b < a$ B. $b < c < a$ C. $b < a < c$ D. $a < b < c$

题型二: 诱导公式

8. (2025·广东模拟) 若 $\alpha \in (0, \frac{\pi}{2})$, 且 $\cos(2\alpha + \frac{\pi}{2}) = \frac{\cos(\frac{\pi}{2} - \beta)}{\sin(\frac{\pi}{2} + \beta)}$, 则 $\frac{5 - \cos^2\alpha}{\tan\beta}$ 的最大值为

- ()
A. $-\frac{\sqrt{2}}{5}$ B. $-\frac{\sqrt{5}}{5}$ C. $-2\sqrt{5}$ D. $-\frac{\sqrt{5}}{10}$





9. (2025·河南模拟) 已知函数 $f(x) = (x-a)^2 \cos(x+\varphi)$ ($0 < \varphi < \frac{\pi}{2}$) 满足 $f(x+2) = -f(-x)$, 则 $[\sin(\varphi+a)]^{2025}$ 的值 ()

- A. -2 B. -1 C. 1 D. 2

10. 已知函数 $f(x) = \sin\left(2x + \frac{n\pi}{2}\right) + 2\cos^2 x$, $n \in \mathbb{Z}$, 则下列说法正确的是 ()

- A. 当 $n=1$ 时, $f(x)$ 图象的一个对称中心为 $\left(\frac{3\pi}{4}, 1\right)$
B. 当 n 为奇数时, $f(x)$ 的最小正周期是 π
C. 当 n 为偶数时, $f(x)_{\max} = 1 + \sqrt{2}$
D. 当 n 为偶数时, $f(x)$ 在 $\left(\frac{\pi}{8}, \frac{3\pi}{8}\right)$ 上单调递减

11. (2015·重庆) 若 $\tan \alpha = 2 \tan \frac{\pi}{5}$, 则 $\frac{\cos\left(\alpha - \frac{3\pi}{10}\right)}{\sin\left(\alpha - \frac{\pi}{5}\right)} = ()$

- A. 1 B. 2 C. 3 D. 4

12. (2014·新课标 I) 设 $\alpha \in \left(0, \frac{\pi}{2}\right)$, $\beta \in \left(0, \frac{\pi}{2}\right)$, 且 $\tan \alpha = \frac{1+\sin \beta}{\cos \beta}$, 则 ()

- A. $3\alpha - \beta = \frac{\pi}{2}$ B. $3\alpha + \beta = \frac{\pi}{2}$ C. $2\alpha - \beta = \frac{\pi}{2}$ D. $2\alpha + \beta = \frac{\pi}{2}$





13. 在斜 $\triangle ABC$ 中, 若 $\sin A = \cos B$, 则 $3\tan B + \tan C$ 的最小值为 ()
- A. $\sqrt{2}$ B. $\sqrt{5}$ C. $\sqrt{6}$ D. $4\sqrt{3}$

题型三: 和差公式

14. (2024·新高考 II) 已知 α 为第一象限角, β 为第三象限角, $\tan \alpha + \tan \beta = 4$, $\tan \alpha \tan \beta = \sqrt{2} + 1$, 则 $\sin(\alpha + \beta) =$ _____.

15. (2024·新高考 I) 已知 $\cos(\alpha + \beta) = m$, $\tan \alpha \tan \beta = 2$, 则 $\cos(\alpha - \beta) =$ ()
- A. $-3m$ B. $-\frac{m}{3}$ C. $\frac{m}{3}$ D. $3m$

16. (2025·河南模拟) 已知 $\alpha, \beta \in (0, \pi)$, 且 $\cos \alpha = \frac{\sqrt{5}}{5}$, $\sin(\alpha + \beta) = \frac{\sqrt{2}}{10}$, 则 $\cos \beta =$ ()
- A. $\frac{\sqrt{10}}{10}$ B. $-\frac{\sqrt{10}}{10}$ C. $\frac{9\sqrt{10}}{50}$ D. $-\frac{9\sqrt{10}}{50}$

17. (2022·新高考 II) 若 $\sin(\alpha + \beta) + \cos(\alpha + \beta) = 2\sqrt{2}\cos(\alpha + \frac{\pi}{4})\sin \beta$, 则 ()
- A. $\tan(\alpha - \beta) = 1$ B. $\tan(\alpha + \beta) = 1$ C. $\tan(\alpha - \beta) = -1$ D. $\tan(\alpha + \beta) = -1$





18. (2012·江苏) 设 α 为锐角, 若 $\cos(\alpha + \frac{\pi}{6}) = \frac{4}{5}$, 则 $\sin(2\alpha + \frac{\pi}{12})$ 的值为_____.

19. (2025·广东模拟) 已知 $\beta \in (0, \frac{\pi}{2})$, 且 $3\sin\alpha = \sin(2\beta - \alpha)$, 则 $\tan\alpha$ 的最大值为 ()

A. $-\frac{\sqrt{2}}{4}$

B. $\frac{\sqrt{2}}{4}$

C. $-\frac{\sqrt{3}}{4}$

D. $\frac{\sqrt{3}}{4}$

题型四: 积化和差、和差化积

20. (2025·南通模拟) 已知 $x, y \in (0, \frac{\pi}{2})$, $\cos(x + y) = -\frac{5}{13}$, $\sin 2x - \sin 2y = -\frac{6}{13}$, 则 $\tan 2x =$ ()

A. $\frac{63}{16}$

B. $\frac{33}{16}$

C. $-\frac{33}{56}$

D. $-\frac{63}{56}$

21. (2025·绵阳模拟) 已知 $\alpha \in (0, \frac{\pi}{2})$, $\beta \in (-\frac{\pi}{2}, 0)$, 且满足 $\cos(\alpha + \beta) = \frac{1}{2}$, 则 $\sin 2\alpha \sin 2\beta$ 最小值为 ()

A. $-\frac{\sqrt{2}}{2}$

B. $-\frac{\sqrt{2}}{4}$

C. $-\frac{1}{2}$

D. $-\frac{1}{4}$





22. 已知 $\cos(\alpha + \beta) = -\frac{1}{3}$, $\cos\alpha + \cos\beta = 1$, 则 $\cos\frac{\alpha-\beta}{2} \cdot \cos\frac{\alpha+\beta}{2} = \underline{\hspace{2cm}}$, $\frac{\sin(\alpha+\beta)}{\sin\alpha + \sin\beta} = \underline{\hspace{2cm}}$.

23. 已知 $f(\theta) = \cos 4\theta + \cos 3\theta$, 且 $\theta_1, \theta_2, \theta_3$ 是 $f(\theta)$ 在 $(0, \pi)$ 内的三个不同零点, 则 ()

A. $\frac{\pi}{7} \in \{\theta_1, \theta_2, \theta_3\}$

B. $\theta_1 + \theta_2 + \theta_3 = \pi$

C. $\cos\theta_1 \cos\theta_2 \cos\theta_3 = -\frac{1}{8}$

D. $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = \frac{1}{2}$

题型五: 正弦平方差+余弦平方差

24. 已知 $\sin\left(\frac{\pi}{4} + \alpha\right)\sin\left(\frac{\pi}{4} - \alpha\right) = \frac{1}{6}$, $\alpha \in \left(\frac{\pi}{2}, \pi\right)$, 则 $\sin 4\alpha = \underline{\hspace{2cm}}$.

25. 求值: (1) $\sin 20^\circ \cos 70^\circ + \sin 10^\circ \sin 50^\circ = \underline{\hspace{2cm}}$;

(2) $\sin 15^\circ \sin 30^\circ \sin 75^\circ = \underline{\hspace{2cm}}$.





26. (2025·丽江模拟) 已知 $\sin(\alpha + \beta) = \frac{1}{2}$, $\sin(\alpha - \beta) = \frac{1}{3}$, 则 $\cos^2\alpha - \cos^2\beta = (\quad)$

A. $\frac{1}{36}$

B. $-\frac{1}{36}$

C. $\frac{1}{6}$

D. $-\frac{1}{6}$





课堂总结



作业

1. 设 θ 为第二象限角, 若 $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2}$, 则 $\sin\theta \cdot \cos\theta =$ _____.
2. 已知 $\tan\alpha = 2$. 求 $\frac{\sin 2\alpha}{\sin^2\alpha + \sin\alpha\cos\alpha - \cos 2\alpha - 1}$ 的值.
3. 已知 $\sin\left(\alpha + \frac{\pi}{3}\right) = \frac{1}{3}$, $\alpha \in \left(\frac{\pi}{2}, \pi\right)$, 则 $\sin\alpha =$ _____.
4. (2011·浙江) 若 $0 < \alpha < \frac{\pi}{2}$, $-\frac{\pi}{2} < \beta < 0$, $\cos\left(\frac{\pi}{4} + \alpha\right) = \frac{1}{3}$, $\cos\left(\frac{\pi}{4} - \frac{\beta}{2}\right) = \frac{\sqrt{3}}{3}$, 则 $\cos\left(\alpha + \frac{\beta}{2}\right) =$ ()
A. $\frac{\sqrt{3}}{3}$ B. $-\frac{\sqrt{3}}{3}$ C. $\frac{5\sqrt{3}}{9}$ D. $-\frac{\sqrt{6}}{9}$
5. 已知 $\sin x - \sin y = -\frac{2}{3}$, $\cos x - \cos y = \frac{2}{3}$, 且 x, y 为锐角, 则 $\tan(x - y) =$ ()
A. $\frac{2\sqrt{14}}{5}$ B. $-\frac{2\sqrt{14}}{5}$ C. $\pm \frac{2\sqrt{14}}{5}$ D. $\pm \frac{5\sqrt{14}}{28}$

